Progression in calculations

Year 1 – Year 6*

NB. Users should familiarise themselves with the introduction (pp 2-10) to this document before referring to individual year group guidance.

**Progression guidance is not provided for EYFS/Reception since the focus should be on the understanding of early number concepts and number sense through the use of concrete manipulatives, as exemplified in the programmes of study.*

Mathematics Mastery

Introduction

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This document outlines the different calculation strategies that should be taught and used in Years 1 to 6, in line with the requirements of the 2014 Primary National Curriculum.

Background

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what pupils should achieve.

One of the key differences is the level of detail included, indicating what pupils should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered.

For example, in Year 2, it is suggested that pupils should be able to ‘add and subtract one-digit and two-digit numbers to 20, including zero’ and a few years later, in Year 5, they should be able to ‘add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)’.

In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils’ calculation skills, and the expectation of using formal methods is rightly coupled with the explicit requirement for pupils to use multiple representations, including concrete manipulatives and images or diagrams – a key component of the mastery approach.

Purpose

The purpose of this document is threefold. Firstly, in this introduction, it outlines the structures for calculations, which enable teachers to systematically plan problem contexts for calculations to ensure pupils are exposed to both standard and non-standard problems. Secondly, it makes teachers aware of the strategies that pupils are formally taught within each year group, which will support them to perform mental and written calculations. Finally, it supports teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding.

The policy only details the strategies; teachers must plan opportunities for pupils to apply these, for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

How to use the document

For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. Please note that the concrete and pictorial representation examples are not exhaustive, and teachers and pupils may well come up with alternatives. The purpose of using multiple representations is to give pupils a deep understanding of a mathematical concept and they should be able to work with and explain concrete, pictorial and abstract representations, and explain the links between different representations. Depth of understanding is achieved by moving between these representations. For example, if a child has started to use a pictorial representation, it does not mean that the concrete cannot be used alongside the pictorial. If a child is working in the abstract, depth can be evidenced by asking them to exemplify their abstract working using a concrete or pictorial representation and to explain what they have done using the correct mathematical vocabulary; language is, of course, one abstract representation but is given particular significance in the national curriculum.

Mathematical language

The 2014 National Curriculum is explicit in articulating the importance of pupils using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for pupils to extend their language around certain concepts.

> "The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof."
>
> *2014 Maths Programme of Study*

Suggested language structures accompany each strategy outlined in this document. These build on one another systematically, which supports pupils in making links between and across strategies as they progress through primary school. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, manipulatives, pictures or diagrams) and explained precisely. High expectations of the mathematical language used are essential, with teachers modelling accurate mathematical vocabulary and expecting pupils' responses to include it ***in full sentences.***

✓	✗
ones	units
is equal to	equals / makes
zero	oh (the letter O)

Presentation of calculations

You will see that throughout this document, calculations are presented in a variety of ways. It is important for pupils' mathematical understanding to experience and work with calculations and missing numbers in different positions relative to the = symbol. Examples used in classwork and independent work should reflect this.

Estimation

Pupils are expected to use their developing number sense from Year 1 to make predictions about the answers to their calculations. As their range of mental strategies increases, these predictions and, later, estimates should become increasingly sophisticated and accurate. All teaching of calculation should emphasise the importance of making and using these estimates to check, first, the sense and, later, the accuracy of their calculations.

Developing number sense

Fluency in arithmetic is underpinned by a good sense of number and an ability to understand numbers as both a concept (e.g. 7 is the value assigned to a set of seven objects) and as something resulting from a process (three beads and four more beads gives seven beads altogether or $3 + 4 = 7$). Understanding that a number can be partitioned in many ways (e.g. $7 = 4 + 3$; $5 + 2 = 7$; $1 + 6 = 7$) is key to being able to use numbers flexibly in calculating strategies. The part-whole model and, later, bar models, are particularly useful for developing a relational understanding of number. Pupils who are fluent in number bonds (initially within ten and then within twenty) will be able to use the 'Make ten' strategy efficiently, enabling them to move away from laborious and unreliable counting strategies, such as 'counting all' and 'counting on'. Increasing fluency in efficient strategies will allow pupils to develop flexible and interlinked approaches to addition and subtraction. At a later stage, applying multiplication and division facts, rather than relying on skip-counting, will continue to develop flexibility with number.

Structures and contexts for calculations

> *"In a technological age, in which most calculations are done on machines, it surely cannot be disputed that knowing which calculation to do is more important than being able to do the calculation."*
>
> Derek Haylock (2014); Mathematics Explained for Primary Teachers, p.56

There are multiple contexts (the word problem or real-life situation, within which a calculation is required) for each mathematical operation (i.e. addition) and, as well as becoming fluent with efficient calculating strategies, pupils also need to become fluent in identifying which operations are required. If they are not regularly exposed to a range of different contexts, pupils will find it difficult to apply their understanding of the four operations. For each operation, a range of contexts can be identified as belonging to one of the conceptual 'structures' defined below.

The **structure** is distinct from both the **operation** required in a given problem and the **strategy** that may be used to solve the calculation. In order to develop good number sense and flexibility when calculating, children need to understand that many strategies (preferably the most efficient one <u>for them</u>!) can be used to solve a calculation, once the correct operation has been identified. There is often an implied link between the given structure of a problem context and a specific calculating strategy. Consider the following question: A chocolate bar company is giving out free samples of their chocolate on the street. They began the day with 256 bars and have given away 197. How many do they have remaining? The reduction context implicitly suggests the action of 'taking away' and might lead to a pupil, for example, counting back or using a formal algorithm to subtract 197 from 256 (seeing the question as $256 - 197 = \square$). However, it is much easier to find the difference between 197 and 256 by adding on (seeing the question as $197 + \square = 256$). Pupils with well-developed number sense and a clear understanding of the inverse relationship between addition and subtraction will be confident in manipulating numbers in this way.

Every effort is made to include multiple contexts for calculation in the Mathematics Mastery materials but, when teachers adapt the materials (which is absolutely encouraged), having an awareness of the different structures and being sure to include a range of appropriate contexts, will ensure that pupils continue to develop their understanding of each operation. The following list should not be considered to be exhaustive but defines the structures (and some suggested contexts) that are specifically included in the statutory objectives and the non-statutory guidance of the national curriculum. Specific structures and contexts are introduced in the Mathematics Mastery materials at the appropriate time, according to this guidance.

Importance of knowns vs unknowns and using part-whole understanding

One of the key strategies that pupils should use to identify the correct operation(s) to solve a given problem (in day-to-day life and in word problems) is to clarify the known and unknown quantities and identify the relationships between them. Owing to the inverse relationship between addition and subtraction, it is better to consider them together as 'additive reasoning', since changing which information is unknown can lead to either addition or subtraction being more suitable to calculate a solution for the same context. For the same reason, multiplication and division are referred to as 'multiplicative reasoning'. Traditionally, approaches involving key vocabulary have been the main strategy used to identify suitable operations but owing to the shared underlying structures, key words alone can be ambiguous and lead to misinterpretation (see for example the question below about Samir and Lena, where the key word 'less' might be identified, but addition is required to solve the problem).

A more effective strategy is to encourage pupils to establish what they know about the relationship between the known and unknown values and if they represent a part or the whole in the problem, supported through the use of part-whole models and/or bar models. In the structures exemplified below, the knowns and unknowns have been highlighted. Where appropriate, the part-whole relationships have also been identified. Pupils should always be given opportunities to identify and discuss these, both when calculating and when problem-solving.

Standard and non-standard contexts

Using key vocabulary as a means of interpreting problems is only useful in what are in this document defined as 'standard' contexts, i.e. those where the language is aligned with the operation used to solve the problem. Take the following example:

First there were 12 people on the bus. Then three ***more*** *people got on. How many people are on the bus now?*

Pupils would typically identify the word 'more' and assume from this that they need to add the values together, which in this case would be the correct action. However, in non-standard contexts, identifying key vocabulary is unhelpful in identifying a suitable operation. Consider this question:

First there were 12 people on the bus and then some more people got on at the school. Now there are 15 people on the bus. How many people got on at the school?

Again the word 'more' would be identified, and a pupil may then erroneously add together 12 and 15. It is therefore much more helpful to consider known and unknown values and the relations between them.

Overexposure to standard contexts and lack of exposure to non-standard contexts will mean pupils are more likely to rely on 'key vocabulary' strategies, as they see that this works in most of the cases they encounter. It is therefore important, when adapting lesson materials, that non-standards contexts are used systematically, alongside standard contexts.

Additive reasoning

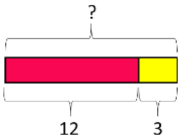
Change structures

augmentation (increasing)

where an existing value has been added to

Standard

First there were 12 people on the bus. Then three more people got on. How many people are on the bus now?

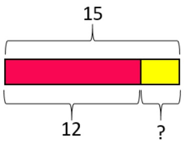


"I know both parts. My first part is twelve and my second part is three. I don't know the whole. I need to add the parts of twelve and three to find the whole."

12 + 3 = ?

Non-standard

First there were 12 people on the bus and then some more people got on at the school. Now there are 15 people on the bus. How many people got on at the school?

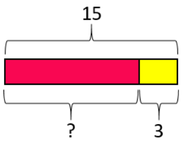


"I know my first part is twelve and I know the whole is 15. I don't know the value of the second part. To find the second part, I could add on from 12 to make 15 or I could subtract 12 from 15."

12 + ? = 15 15 – 12 = ?

Non-standard

First there were some people on the bus then it stopped to pick up three more passengers at the bank. Altogether now there are 15 people on the bus. How many were people were on the bus before it stopped at the bank?



"I know the value of the second part is three and that the whole is 15. I don't know the value of the first part. To find the first part, I could add on from three to make 15 or I could subtract three from 15."

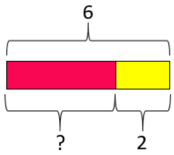
? + 3 = 15 15 – 3 = ?

reduction (decreasing)

where an existing value has been reduced

Standard

First Kieran had six plates in his cupboard. Then he took two plates out to use for dinner. How many plates are in the cupboard now?

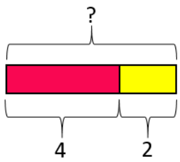


"I know the whole is six. I know one of the part that has been taken away is two. I don't know the other part. I need to subtract the known part, two, from the whole, six, to find the remaining part."

6 – 2 = ? 2 + ? = 6

Non-standard

First there were some plates in the cupboard. Then Kieran took two out for dinner. Now there are four left. How many plates were in the cupboard to start with?

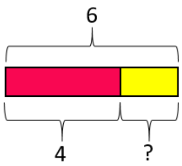


"I know the part that has been taken away is two and the part that is left is four. I don't know the whole. I can find the whole by adding the parts of four and two."

? – 2 = 4 2 + 4 = ?

Non-standard

First there were six plates in the cupboard. Then Kieran took some out for dinner. There are now four plates left in the cupboard. How many did Kieran take out?



"I know the whole is six and the remaining part is four. I don't know the part that was taken away. To find the part that was taken away I can add on from four to make six or I could subtract four from six."

6 – ? = 4 6 – 4 = ?

Note: the 'first... then... now' structure is used heavily in KS1 to scaffold pupils' understanding of change structures. Once pupils are confident with the structures, such linguistic scaffolding can be removed, and question construction can be changed to expose pupils to a greater range of nuance in interpreting problems. For example, the second and third reduction problems could be reworded as follows:

Kieran took two plates out of his cupboard for dinner. There were four left. How many plates were in the cupboard to begin with?

There were six plates in the cupboard before Kieran took some out for dinner. If there were four plates left in the cupboard, how many did Kieran take out?

These present the same knowns and unknowns, and therefore the same bar models and resulting equations to solve the problems; however, the change in wording makes them more challenging to pupils who have only worked with a 'first... then... now' structure so far.

Part-whole structures

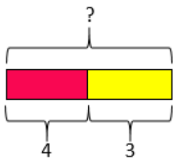
Combination (aggregation)/partitioning

combining two or more discrete quantities/splitting one quantity into two or more sub-quantities

Hakan and Sally have made a stack of their favourite books. Four books belong to Hakan, three to Sally. How many books are in the stack altogether?

"I know both parts. One part is four and the other part is three. I don't know the whole. I need to add the parts of three and four to find the whole."

4 + 3 = ?　　3 + 4 = ?

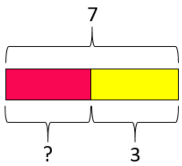


(Only one problem has been written for combination as, owing to the commutativity of addition, the only change in question wording would be to swap Hakan and Sally's names. The resulting bar model and calculation would be identical.)

Sally and Hakan have made a stack of their favourite books. There are seven books altogether. If three of them are Sally's, how many belong to Hakan?

"I know the whole is seven and that one of the parts is three. I don't know the other part. I need to add on from three to make seven or subtract three from seven to find the other part."

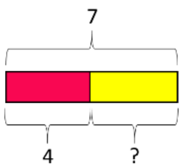
3 + ? = 7　　7 – 3 = ?



Sally and Hakan have made a stack of their favourite books. There are seven books altogether. If four of them are Hakan's, how many belong to Sally?

"I know the whole is seven and that one of the parts is four. I don't know the other part. I need to add on from four to make seven or subtract four from seven to find the other part."

4 + ? = 7　　7 – 4 = ?



Note: all part-whole contexts are considered to be 'standard', as the language of part-whole is unambiguous.

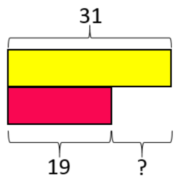
Comparison structures

Comparison structures involve a relationship between two quantities; their relationship is expressed as a difference. The structures vary by which of the values are known/unknown (the larger quantity, the smaller quantity and/or their difference). Part-whole language is not used here because the context contains not one single 'whole', but instead two separate quantities and it is the relationship between them being considered. Comparison bar models are therefore used to model these structures, which are known to be the most challenging for pupils to interpret.

Smaller quantity and larger quantity are known (comparative difference)

Standard

Navin has saved £19 from his pocket money. Sara has saved £31 from her pocket money. How much **more** has Sara saved than Navin? ***or*** How much **less** has Navin saved than Sara?



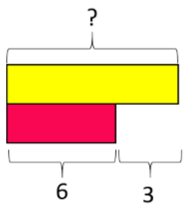
"I know one quantity is 19 and the other quantity is 31. I don't know the difference. To find the difference I could add on from 19 to make 31 or I could subtract 19 from 31."

19 + ? = 31 31 – 19 = ?

Smaller quantity and difference are known (comparative addition)

Standard

Ella has six marbles. Robin has three **more** than Ella. How many marbles does Robin have?

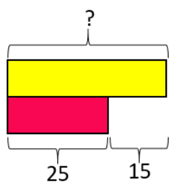


"I know the smaller quantity is six. I know the difference is three. I don't know the larger quantity. To find the larger quantity I need to add three to six."

6 + 3 = ?

Non-standard

Samir and Lena are baking shortbread but Lena's recipe uses 15g **less** butter than Samir's. If Lena needs to use 25g of butter, how much does Samir need?



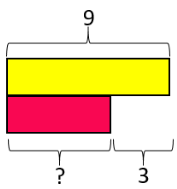
"I know the smaller quantity is 25. I know the difference between the quantities is 15. I don't know the larger quantity. To find the larger quantity I need to add 15 to 25."

? - 15 = 25 25 + 15 = ?

Larger quantity and difference are known (comparative subtraction)

Non-standard

Ella has some marbles. Robin has three **more** than Ella and he has nine marbles in total. How many marbles does Ella have?

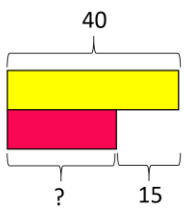


"I know the larger quantity is nine. I know the difference between the quantities is three. I don't know the smaller quantity. To find the smaller quantity I need to add on from three to make nine or subtract three from nine."

? + 3 = 9 9 – 3 = ?

Standard

Samir's shortbread recipe uses 40g of butter. Lena's recipe uses 15g **less** butter. How much butter does Lena need?



"I know one quantity is 40. I know the difference between the quantities is 15. I don't know the smaller quantity but I know it is 15 less than 40. To find the smaller quantity, I need to subtract 15 from 40."

40 – 15 = ? ? + 15 = 40

Mathematics Mastery

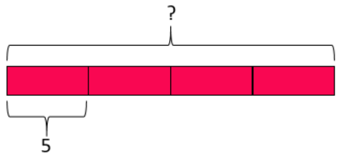
Multiplicative reasoning

Repeated grouping structures

repeated addition

groups (sets) of equal value are combined or repeatedly added

There are four packs of pencils. Each contains five pencils. How many pencils are there?



"I know there are four equal parts and that each part has a value of five. I don't know the value of the whole. To find the whole, I need to multiply four and five."

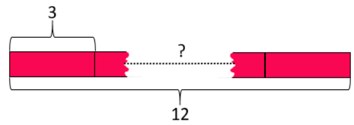
5 + 5 + 5 + 5 = ?

5 × 4 = ?

repeated subtraction (grouping)

groups (sets) of equal value are partitioned from the whole or repeatedly subtracted

There are 12 counters. If each child needs three counters to play the game, how many children can play?



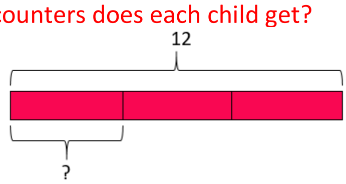
"I know the whole is twelve and that the value of each equal part is three. To find the number of equal parts, I need to know how many threes are in twelve."

3 × ? = 12   12 ÷ 3 = ?

sharing (into equal groups)

the whole is shared into a known number (must be a positive integer) of equal groups (sets)

Share twelve counters equally between three children. How many counters does each child get?



"I know the whole is twelve and the number of equal parts is three. I don't know the value of each part. To find the value of each part, I need to know what goes into twelve three times."

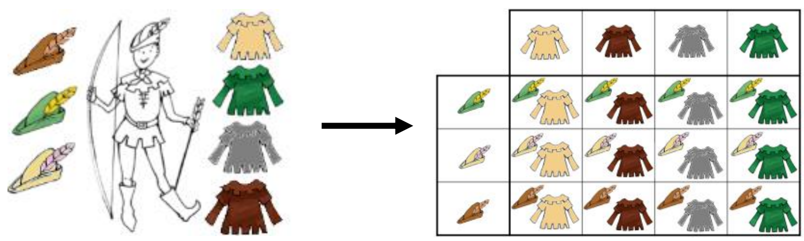
? × 3 = 12   12 ÷ 3 = ?

Cartesian product of two measures

correspondence

calculating the number of unique combinations that can be created from two (or more) sets

Robin has three different hats and four different tops. How many different outfits can he create, that combine one hat and one top?



"I know how many hats there are and I know how many tops there are. I don't know the number of different outfits that can be created. To find the number of outfits, I need to find how many different tops can be worn with each hat or how many different hats can be worn with each top."

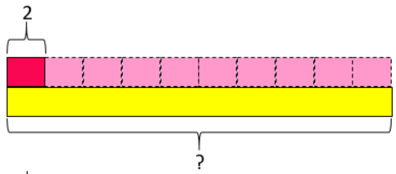
4 × 3 = ?   3 × 4 = ?

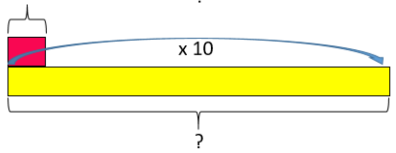
Scaling structures

scaling up
(‘times greater/times as much’)

the original value is increased by a given scale factor

Rita receives £2 pocket money every week. Sim earns ten times as much money for her paper round. How much money does Sim earn?





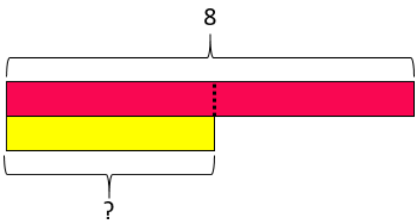
“I know one value is two and I know the second value is ten times greater. I don’t know the second value. To find the second value, I need to multiply two by ten.”

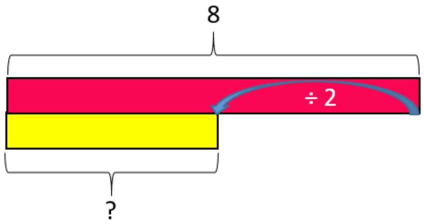
2 × 10 = ?

scaling down
(‘times smaller/times less’)

the original value is reduced by a given scale factor

The house in my model village needs to be half the height of the church. If the church is 8 cm tall, how tall does the house need to be?





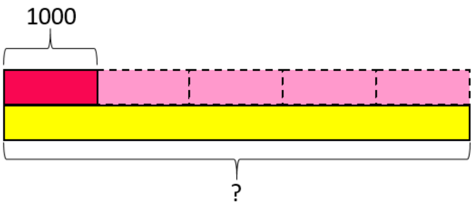
“I know one value is eight and I know the second value is half as great. I don’t know the second value. To find the second value, I need to halve eight (or divide it by two).”

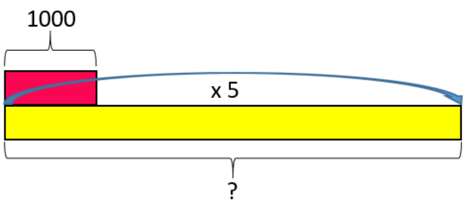
Half of 8 is ? 8 ÷ 2 = ?

scaling up (‘times as many’)

the value of the original quantity is increased by a given scale factor

The Albert Hall can hold five times as many people as the Festival Hall. If the Festival Hall holds 1000 people, how many does the Albert Hall hold?





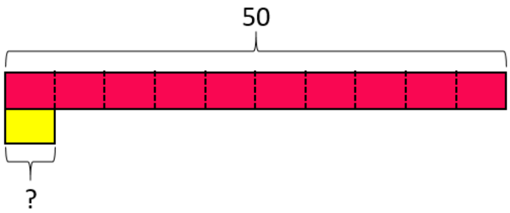
“I know one value is 1000 and I know the second value is five times greater. I don’t know the second value. To find the second value, I need to multiply 1000 by five.”

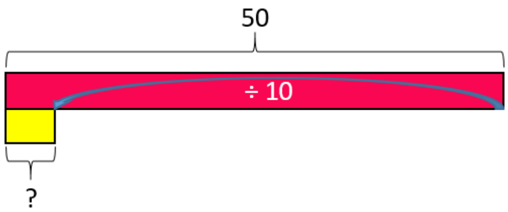
1000 × 5 = ?

scaling down (‘times fewer’)

the value of the original quantity is decreased by a given scale factor

Anouska’s garden pond has ten times fewer frogs than fish. If there are fifty fish, how many frogs are there?





“I know one value is 50 and I know the second value is ten times less. I don’t know the second value. To find the second value, I need to divide fifty by ten.”

50 ÷ 10 = ?